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Micro-mechanical behaviors of SMA composite materials under hygro-thermo-elastic strain fields

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Abstract

An analytical procedure to evaluate the behavior of shape memory alloy (SMA) composite under hygrothermal environment is presented. The SMA wires are considered as inclusions embedded in a homogeneous matrix medium of the composite. The inhomogeneity associated with the phase transformation and thermal strains in the SMA wire as well as the hygrothermal strain in the matrix is homogenized using Eshelby's equivalent inclusion method. In the present work, a similar approach adopted for SMA composites by Marfia and Sacco [Marfia, S., Sacco, E., 2005. Micromechanics and homogenisation of SMA-wire-reinforced materials. J. Appl. Mech. 72 (2), 259–268.] is considered in order to validate the response of SMA composite subjected to thermo-elastic strain field. However, in the present approach, certain modifications and new derivations for the inelastic strain tensors is carried out. First, the constitutive laws for the SMA wire and matrix are expressed in terms of the average strain in the composite. The evolutionary equations used to characterize the pseudoelastic (PE) behavior of the SMA wire are redefined in terms of the eigen strains (phase transformation and thermal strains) occurring in the SMA wire, which are then expressed in terms of the average strain in the composite. Further, the SMA composite constitutive law under coupled hygro-thermo-elastic strain fields is proposed. The generic homogenized hygric and thermal inelastic composite tensors required for the proposed hygro-thermo-elastic constitutive law are derived. Finally, the SMA composite lamina is characterized using Eshelby's equivalent inclusion method. Using the proposed modifications and derivations, the analytical results are validated for the case of thermo-elastic strain fields and the procedure is then extended to evaluate the SMA composite behavior under hygro-thermo-elastic strain fields. The results include the effect of thermo-elastic and hygro-thermo-elastic strains on the transformation stresses and the nature of hysteresis due to hygric and thermo-elastic strains.

Keywords: Shape memory alloy; Pseudoelastic effect; Eshelby's method; Inclusion; Eigen strain; SMA composite lamina; Hygro-thermo-elastic strain

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1. Introduction

Smart materials are used in adaptive structures to perform multi-functional tasks. The potential use of shape memory alloys as active fibres, embedded in a composite lamina is one such application to develop SMA embedded composite actuators. In recent years, much interest has been shown through analytical and experimental studies in the area of SMA composites. However, modeling the mechanics of such composites is involved with computation of SMA fibre behavior embedded in a matrix. SMA materials exhibit pseudoelasticity (PE) and shape memory effects due to the applied stress and temperature, respectively. Several constitutive models have been developed to simulate the non-linear phase transformation performance of SMA wire. Most of these constitutive models are phenomenological-based macro scale models, developed for quasi-static loading only. Popular among them is 1D model proposed by Tanaka (1986), in which the second law of thermodynamics is expressed in terms of Helmholtz free energy and Clausius–Duhem inequality. Further, the transformation kinetics is evaluated by expressing the martensite fraction as an exponential function of temperature and stress. Rogers and Liang (1990) have modified Tanaka's model by replacing the exponential transformation kinetics with a cosine law, where the elastic modulus is assumed to be a linear function of the martensite fraction. Brinson (1993) modified the constitutive equation of Tanaka as well as that of Rogers and Liang, and proposed a modeling scheme in which the martensite fraction is divided into two parts; one corresponding to temperature induced and the other related to stress induced. This model besides predicting the stress–strain behavior is also able to simulate the recovery stresses and recovery strains needed for the design of SMA based smart structural systems. James and Dimitris (1996a) proposed a constitutive model based on thermodynamic principles in order to study the damping capacity and actuator efficiency of converting heat into work done. Adiabatic and isothermal phase transformations were compared. The results showed that adiabatic response exhibits greater thermal hardening than isothermal process and the stress required to complete adiabatic transformation is higher than that required for isothermal phase transformations. Auricchio and Sacco (1997a,b, 1999) have developed a one-dimensional constitutive model able to reproduce the superelastic as well as the shape memory effects, taking into account the martensite reorientation. Tadashige et al. (2004) presented a one-dimensional constitutive model for SMA, which considers the inner hysteresis loops under thermo-mechanical loading.

Analytical studies have also been carried out to analyze the pseudoelastic and shape memory effects of SMA wire embedded composite materials. Using micro-mechanics approach of Tanaka, James and Dimitris (1996b) formulated the analysis procedure to evaluate the thermo-elastic response of SMA composites using micro-thermodynamics approach. In order to volume average, the equations for macroscopic eigen strains were derived by performing a localization process in which the mesoscopic fields were derived from the boundary conditions (uniform strain, uniform stress and periodicity conditions) imposed on the SMA composite. However, the Mori–Tanaka micro-mechanics approach for non-dilute composites was used for the homogenization procedure. Victor (1996) reported an analytical procedure based on micro-mechanics concept for SMA composites by adopting a multicell approach under isothermal temperature fields. Two different micro-mechanical approaches; the multicell approach and concentric cylinder approach were proposed to evaluate the equivalent maximum transformation strain and local stresses in the fibre and matrix, respectively. In the analytical procedure, the SMA composite constitutive model was not described and analytical results for the predicted response were not presented. In the analytical modeling procedure presented by Cherkaoui et al. (2000), the macroscopic constitutive relations of the composite were established by using self-consistent approach where the micro–macro relationships was established by volume averaging and by introducing the concept of stress and strain concentration tensors. In this micro-mechanics modeling, the SMA wires were considered as spherical inclusions randomly distributed and not as axial continuous cylinder fibres. The effective properties were also not evaluated. A thermo-elastic constitutive model and a finite element formulation was developed by Travis (2000) for predicting the thermo-mechanical response of uniaxial SMA hybrid composite structures subjected to combined thermal and mechanical loads. An engineering (rule of mixtures) approach was used to derive the mechanical properties of a SMAHC lamina in terms of the properties of the constituent materials. As a result, the effect of fibre geometry was not included in the analytical model and also the basic composite transformation stresses during the SMA phase transformations were not evaluated. Wei et al. (2003) studied the strain energy absorbing capacity of composites with SMA wires integrated with E-glass epoxy matrix. The results show that with resistive heating significant increase in energy absorption and damping is possible; however, the structural integrity of the smart composite with embedded SMA material needs to be improved. For the tests conducted at room temperature (martensite phase) up to 50% increase in strain energy absorption was observed. For the tests conducted at elevated temperatures (austenite phase) the increase in energy absorption was as high as 600% compared to that for the baseline structure. This demonstrates potential applications of SMA composites for crash-safety in automotive engineering and landing gears of aircraft. However, analytical modeling was not presented to evaluate the homogenized properties as well as the composite stress–strain behaviors. Marfia and Sacco (2005) proposed analytical model to evaluate the homogenized SMA composite properties as well as the stress–strain behaviors using Eshelby's equivalent inclusion method. The results were evaluated for thermo-elastic loading conditions. However, some inconsistency was observed in the strain tensors $(\overline{P})$ and $(\overline{t})$, which have been redefined in the present work. Further, the analysis results did not include the hygric effects on the behavior of SMA composite.

Although several SMA wire constitutive models have been developed, very few micro-mechanical analytical models for SMA composites are presented, which are not easily used in practice because they are qualitative in nature and have not been experimentally verified. Hence, no model has been available for broad use by the research and engineering communities. An alternative approach is to employ a composite model that can be used for experimental measurement of fundamental engineering properties and also to predict analytically the transformation stresses as well as the associated hysteresis under hygro-thermo-elastic loading conditions. This type of model should also be amenable to incorporation into general structural analysis tools.

1.1. Motivation

In general SMA has been employed in a one-dimensional wire form (James and Dimitris, 1996b; Victor, 1996; Travis, 2000; Wei et al., 2003; Marfia and Sacco, 2005) to develop smart structure concepts for shape control and damping applications. A well-trained SMA wire can be easily loaded or unloaded either through temperature (by applying current) or by mechanical means (by applying force) to induce the forward or reverse phase transformations. However, loading and unloading are the critical issues for SMA composites with the SMA wires embedded in a polymer matrix. The elastic properties of SMA will change during its phase transformation cycle, so would the performance of the active system (SMA lamina). Therefore, a correct estimation of elastic moduli of SMA composite would help to design the composite smart structures, where the potential energy imparted by SMA is accurately computed. Generally, SMA wire will have a wide range of austenite finish temperatures (−50 °C to 100 °C). When SMA is used in aircraft composite structures, the effect of moisture may influence the transformation stresses of SMA and its actuation efficiency. As a result, the analysis has to include such environmental effects on SMA actuation. Since, SMA wires are usually embedded in a polymer type matrix, which absorb moisture; the effect of moisture on the behavior of whole SMA lamina under the hygrothermal loading conditions is important. The effect of hygrothermal fields on the transformation stresses, overall hysteresis and micro-mechanical response of SMA embedded composites has not been addressed so far. Hence, this issue is analyzed in the present work. Since, the areas of SMA composite applications depend directly on the damping performance, an analytical approach is proposed to compute the transformation stresses and analyze the hysteresis behavior of SMA composite under the influence of elevated moisture and temperature conditions. Micro-mechanics based homogenization technique presented by Mura (1982) is applied using dilute distribution theory to arrive at an expression for the total strain in the SMA wire in terms of the total average strain in the composite. This modification is required because the SMA wire is embedded within the matrix and the strain to induce the phase transformations within the SMA wire should be the total strain field experienced by the composite. A constitutive law in terms of coupled hygro-thermo-elastic strain field is proposed to compute the thermo-elastic as well as hygro-thermo-elastic transformation stresses of the SMA composite. In the present procedure, three inelastic composite strain tensors, namely, the strain tensor $(\overline{\varepsilon}_{\mathrm{p}})$ due to phase transformation strains in the SMA wire, the strain tensor $(\overline{\varepsilon}_{\mathrm{m}})$ due to hygric strain in the matrix and the strain tensor $(\overline{\varepsilon}_{\mathrm{t}})$ due to thermal strains in the SMA wire and matrix, are formulated. These average strain tensors $\overline{\varepsilon}_{\mathrm{p}}, \overline{\varepsilon}_{\mathrm{t}}$ and $\overline{\varepsilon}_{\mathrm{m}}$ are redefined to derive the phase transformation constant $(\overline{P})$, thermo-elastic constant $(\overline{\lambda})$ and hygro-thermo-elastic constant $(\overline{\Lambda})$ for the SMA composite. The procedure is validated with approach adopted by Marfia and Sacco (2005) to assess the efficiency of redefined strain tensor

($\bar{\varepsilon}_p$) and ($\bar{\varepsilon}_t$) for thermo-elastic strain field of the composite. Finally, the thermo-elastic (TE) and hygro-thermo-elastic (HTE) hysteretic behaviors of SMA composite for different SMA wire volume fractions under quasi-static strain increments along the SMA wire direction (monotonic loading) are simulated and the analytical results are presented. The work also evaluates the on-axis as well as the off-axis elastic properties of the SMA composite lamina considering the aspect ratio of the fibres. The proposed modeling procedure can be easily implemented in a numerical scheme (finite element method) for the analysis of laminated SMA composite smart structures. Further, the procedure can be used for the design and experimental validation of the SMA lamina/laminate smart structures under hygro-thermo-elastic conditions.

2. Homogenization procedure for the SMA composite

The disturbance in an applied stress field due to an inhomogeneous inclusion (Ω) whose elastic moduli is different from that of matrix (M) can be simulated by an eigen stress caused by an eigen strain in a homogeneous matrix medium. The average stress ($\bar{\sigma}$) and strain ($\bar{\varepsilon}$) in the composite (Mura, 1982) are

$$\bar{\sigma} = \sigma^{\Omega}\vartheta^{\Omega} + \sigma^{M}\vartheta^{M} \tag{1}$$

$$\bar{\varepsilon} = \varepsilon^{\Omega}\vartheta^{\Omega} + \varepsilon^{M}\vartheta^{M} \tag{2}$$

where ϑ^{Ω} and ϑ^{M} are the volume fractions of the inclusion (SMA wire) and homogeneous matrix domain, respectively.

2.1. Equivalent inclusion method for eigen strains

The equivalent inclusion method is based on the stress consistency between inhomogeneous inclusion and homogeneous matrix surrounding the inclusion. Since, the SMA materials have different elastic moduli and strains from that of the matrix, they can be termed as inclusions with inhomogeneity. In order to obtain the homogenized properties of such a composite medium shown in Fig. 1, the equivalent inclusion method is adopted. The Hooke's law for inclusion and matrix (Mura, 1982) in terms of the applied stress (σ^{Ω}) and induced disturbance stress (σ^{d}) is written as,

$$\sigma^{d} + \sigma^{\Omega} = C^{\Omega}\{\varepsilon^{d} + \varepsilon^{\Omega} - \gamma^{\Omega}\} \tag{3}$$

$$\sigma^{d} + \sigma^{M} = C^{M}\{\varepsilon^{d} + \varepsilon^{\Omega} - \gamma^{\Omega} - \varepsilon^{*}\} \tag{4}$$

where $C^{\Omega}, \varepsilon^{\Omega}$ and γ^{Ω} represent the elastic modulus, the total strain and inelastic strain of the inclusion, and σ^{M}, C^{M} denote the stress and elastic modulus of the matrix, respectively. The parameter ε^{d} is the disturbance strain in the inclusion and matrix, and ε^{*} is the fictitious eigen strain introduced for the homogenization procedure in the matrix material.

The Eshelby's stress consistency equation in terms of an assumed eigen strain (ε^{*}) in the homogeneous matrix medium is expressed as below (Mura, 1982)

$$C^{\Omega}\{\varepsilon^{d} + \varepsilon^{\Omega} - \gamma^{\Omega}\} = C^{M}\{\varepsilon^{d} + \varepsilon^{\Omega} - \gamma^{\Omega} - \varepsilon^{*}\} \tag{5}$$

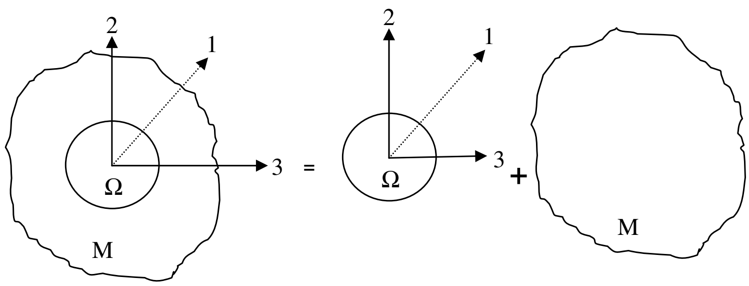

Fig. 1. Inclusion (Ω) and domain (M).

If the applied stress field ($\bar{\sigma}$) is uniform, then according to Eshelby, the corresponding eigen strain in the inclusion results in a uniform disturbance strain within the inclusion. Therefore for Eq. (5), a solution with a uniform eigen strain is

$$\varepsilon^{\mathrm{d}} = S^{\Omega}\{\gamma^{\Omega} + \varepsilon^{*}\} \tag{6}$$

where S^{Ω} is the fourth order Eshelby tensor.

The eigen strain is evaluated by using the Eshelby's equivalent stress consistency equation as

$$\varepsilon^{*} = P^{\Omega}\bar{\varepsilon} + Q^{\Omega}\gamma^{M} \tag{7}$$

where $A^{\Omega} = [(C^{M} - C^{\Omega})^{-1}C^{M}]$.

Substituting Eqs. (6) and (7) into Eq. (5), the strain in the inclusion is derived as

$$\varepsilon^{\Omega} = \gamma^{\Omega} + \mathrm{A}^{\Omega}\{Q^{\Omega}\gamma^{\Omega} + P^{\Omega}\bar{\varepsilon}\} \tag{8}$$

where the variables P^{Ω} and Q^{Ω} are given by the following expressions

$$P^{\Omega} = \{C^{M} - [C^{M} - C^{\Omega}]S^{\Omega}\}^{-1}[C^{M} - C^{\Omega}] \tag{9}$$

$$Q^{\Omega} = [(C^{M} - C^{\Omega})^{-1}C^{M} - S^{\Omega}]^{-1}[S^{\Omega} - I] \tag{10}$$

Therefore, using Eq. (7) the total disturbance strain is evaluated as

$$\varepsilon^{\mathrm{d}} = S^{\Omega}\{\gamma^{\Omega} + P^{\Omega}\bar{\varepsilon} + Q^{\Omega}\gamma^{\Omega}\} \tag{11}$$

Finally, the elastic strain in the inclusion is

$$\varepsilon^{\Omega}_{\mathrm{el}} = A^{\Omega}\{P^{\Omega}\bar{\varepsilon} + Q^{\Omega}\gamma^{\Omega}\} \tag{12}$$

3. Constitutive relations for the SMA wire and elastic matrix

The application of thermo-mechanical behavior of SMA wire is achieved by characterizing the stress–strain behaviors. The constitutive models describing such coupled behaviors are already developed. In the present work, the equivalent inclusion method proposed by Mura (1982) is used to evaluate the SMA composite properties and to derive the strains in the fibre (SMA wire) and matrix in terms of the average strain in the composite. A one-dimensional constitutive model proposed by Marfia and Sacco (2005) is adopted. However, the model is redefined in terms of the eigen strains occurring in the SMA wire (inclusion), which is considered as isotropic material and the phase transformations are assumed to occur along the actuation direction of the SMA wire as well as the SMA composite material.

3.1. Constitutive equation for the SMA wire

For an infinitesimal deformation, the total strain (ε^{Ω}) in the SMA inclusion (Ω) is considered as the sum of elastic strain ($\varepsilon^{\Omega}_{\mathrm{el}}$) and inelastic strain ($\gamma^{\Omega}$), which is given by the following expression

$$\varepsilon^{\Omega} = \varepsilon^{\Omega}_{\mathrm{el}} + \gamma^{\Omega} \tag{13}$$

The inelastic strain (phase transformation and thermal strain) in the SMA wire is given as

$$\gamma^{\Omega} = \xi^{\Omega}_{\mathrm{s}}\varepsilon_{L} + \alpha^{\Omega}(T^{\Omega} - T^{\Omega}_{0}) \tag{14}$$

where $\xi^{\Omega}_{\mathrm{s}}, \varepsilon_{\mathrm{L}}, \alpha^{\Omega}, T^{\Omega}$ and T^{Ω}_{0} are the martensite fraction, residual strain, coefficient of thermal expansion, current temperature and reference temperature for the SMA wire, respectively.

The isotropic stress–strain relationship for the SMA wire is

$$\sigma^{\Omega} = C^{\Omega}(\varepsilon^{\Omega}_{\mathrm{el}}) \tag{15}$$

where σ^{Ω} and C^{Ω} are the stress and fourth order elasticity tensor for the SMA wire, respectively.

Using Eqs. (8) and (13) along with Eq. (15) yields the following expression for the stress in the SMA wire in terms of average strain in the SMA composite

$$\sigma^{\Omega} = C^{\Omega}A^{\Omega}[Q^{\Omega}\gamma^{\Omega} + P^{\Omega}\bar{\varepsilon}] \tag{16}$$

3.1.1. Evolutionary equation for austenite to martensite phase transformations of the SMA wire

The evolutionary equation for the volume fraction of martensite ($\xi_{\mathrm{s}}^{\Omega}$) during austenite to martensite (AS) transformation is

$$\dot{\xi}_{\mathrm{s}}^{\Omega} = -(1-\xi_{\mathrm{s}}^{\Omega})\left[\frac{\dot{\sigma}^{\Omega}}{\sigma^{\Omega} - \tilde{\sigma}_{\mathrm{f}}^{\mathrm{AS}}}\right] + \left[\frac{\dot{T}^{\Omega}}{T^{\Omega} - \widetilde{T}_{\mathrm{f}}^{\mathrm{AS}}}\right] \tag{17}$$

where the parameter $\widetilde{T}_{\mathrm{f}}^{\mathrm{AS}}$ and T^{Ω} are the temperatures at which the austenite to martensite phase transformation is complete and working temperature for the SMA, respectively. The parameter is defined as

$$\widetilde{T}_{\mathrm{f}}^{\mathrm{AS}} = \frac{\sigma^{\Omega} - \sigma_{\mathrm{f}}^{\mathrm{AS}} + C^{\mathrm{AS}}T_{\mathrm{s}}^{\mathrm{AM}}}{C^{\mathrm{AS}}} \tag{18}$$

The activation conditions (switching from elastic region to transformation region) for conversion of the austenite to martensite phase are:

$$\tilde{\sigma}_{\mathrm{s}}^{\mathrm{AS}} \leqslant \sigma^{\Omega} \leqslant \tilde{\sigma}_{\mathrm{f}}^{\mathrm{AS}} \quad \text{and} \quad |\,\dot{\sigma}^{\Omega}\,| > 0 \tag{19}$$

where $\tilde{\sigma}_{\mathrm{s}}^{\mathrm{AS}}$ and $\tilde{\sigma}_{\mathrm{f}}^{\mathrm{AS}}$ are the material parameters, $|.|$ is the absolute value and a superpose dot indicates a time derivative of the absolute value. The material parameter $\tilde{\sigma}_{\mathrm{s}}^{\mathrm{AS}}$ and $\tilde{\sigma}_{\mathrm{f}}^{\mathrm{AS}}$ are the stresses at which the austenite to martensite phase transformation starts and completes, respectively. The parameters are given by the following expressions

$$\tilde{\sigma}_{\mathrm{s}}^{\mathrm{AS}} = \sigma_{\mathrm{s}}^{\mathrm{AS}} + C^{\mathrm{AS}}(T^{\Omega} - T_{\mathrm{s}}^{\mathrm{AM}}), \quad \tilde{\sigma}_{\mathrm{f}}^{\mathrm{AS}} = \sigma_{\mathrm{f}}^{\mathrm{AS}} + C^{\mathrm{AS}}(T^{\Omega} - T_{\mathrm{s}}^{\mathrm{AM}}) \tag{20}$$

where the material parameters C^{AS} and $T_{\mathrm{s}}^{\mathrm{AM}}$ are the Clausius–Clapeyron constant and transformation start temperature, respectively.

The expressions for $\tilde{\sigma}_{\mathrm{f}}^{\mathrm{AS}}$ and $\widetilde{T}_{\mathrm{f}}^{\mathrm{AS}}$ are substituted in the evolutionary Eq. (17) and simplified form is written as follows

$$\dot{\xi}_{\mathrm{s}}^{\Omega} = -(1-\xi_{\mathrm{s}}^{\Omega})\left[\frac{\dot{\sigma}^{\Omega} - \dot{T}^{\Omega}C^{\mathrm{AS}}}{\sigma^{\Omega} - \tilde{\sigma}_{\mathrm{f}}^{\mathrm{AS}}}\right] \tag{21}$$

Eq. (21) is integrated in time to yield and the expression for martensite fraction in terms of the applied stress is

$$\Delta\xi_{\mathrm{s}}^{\Omega} = -(1-\xi_{\mathrm{s}_n}^{\Omega})\left[\frac{(\sigma^{\Omega} - \sigma_n^{\Omega}) - (T^{\Omega} - T_n^{\Omega})C^{\mathrm{AS}}}{(\tilde{\sigma}^{\Omega} - \tilde{\sigma}_{\mathrm{f}}^{\mathrm{AS}}) - [(\sigma^{\Omega} - \sigma_n^{\Omega}) - (T_{\Omega} - T_n^{\Omega})C^{\mathrm{AS}}]}\right] \tag{22}$$

where the parameters with subscript ‘n’ indicate the values of previous loading step.

In order to derive the expression for increment in the volume fraction of single variant martensite in terms of the average strain in the composite, it is required to substitute Eq. (16) into Eq. (22). Upon substitution, the expression for the volume fraction of single variant martensite during austenite to martensite transformation is

$$\Delta\xi_{\mathrm{s}}^{\Omega} = -(1-\xi_{\mathrm{s}_n}^{\Omega})\left\{\frac{B^{\Omega} - (T^{\Omega} - T_n^{\Omega})C^{\mathrm{AS}}}{[C^{\Omega}\mathrm{A}^{\Omega}(Q^{\Omega}\gamma^{\Omega} + P^{\Omega}\bar{\varepsilon}) - \tilde{\sigma}_{\mathrm{f}}^{\mathrm{AS}}] - [(B^{\Omega}) - (T^{\Omega} - T_n^{\Omega})C^{\mathrm{AS}}]}\right\} \tag{23}$$

where $B^{\Omega} = C^{\Omega}\mathrm{A}^{\Omega}[(Q^{\Omega}\gamma^{\Omega} + P^{\Omega}\bar{\varepsilon}) - (Q^{\Omega}\gamma_n^{\Omega} + P^{\Omega}\bar{\varepsilon}_n)]$

3.1.2. Evolutionary equation for martensite to austenite phase transformations of the SMA wire

The expression for the volume fraction of martensite during the reverse phase transformation from martensite to austenite (SA) is

$$\dot{\xi}_{\rm s}^{\Omega} = (\xi_{\rm s}^{\Omega})\left[\frac{\dot{\sigma}^{\Omega}}{\sigma^{\Omega} - \tilde{\sigma}_{\rm f}^{\rm SA}} + \frac{\dot{T}^{\Omega}}{T^{\Omega} - \widetilde{T}_{\rm f}^{\rm SA}}\right] \tag{24}$$

The temperature, $\widetilde{T}_{\rm f}^{\rm SA}$, is defined as

$$\widetilde{T}_{\rm f}^{\rm SA} = \frac{(\sigma^{\Omega} + C^{\rm SA}T_{\rm f}^{\rm SA})}{C^{\rm SA}} \tag{25}$$

where $T_{\rm f}^{\rm SA}$ is the temperature at which the martensite to austenite (S–A) phase transformation is complete and $C^{\rm SA}$ is the Clausius–Clapeyron constant for the phase transformation (S–A).

The activation conditions (switching from elastic region to transformation region) for the conversion of martensite to austenite phase are:

$$\tilde{\sigma}_{\rm f}^{\rm SA} \leqslant \sigma^{\Omega} \leqslant \tilde{\sigma}_{\rm s}^{\rm SA} \quad \text{and} \quad |\,\sigma^{\Omega}\,| < 0 \tag{26}$$

where $\tilde{\sigma}_{\rm s}^{\rm SA}$ and $\tilde{\sigma}_{\rm f}^{\rm SA}$ are the material parameters, $|.|$ is the absolute value and a superpose dot indicates a time derivative of the absolute value. The material parameters $\tilde{\sigma}_{\rm s}^{\rm SA}$ and $\tilde{\sigma}_{\rm f}^{\rm SA}$ are the stresses at which the martensite to austenite phase transformation starts and completes, respectively, and are expressed as

$$\tilde{\sigma}_{\rm s}^{\rm SA} = C^{\rm SA}(T^{\Omega} - T_{\rm s}^{\rm SA}), \quad \tilde{\sigma}_{\rm f}^{\rm SA} = C^{\rm SA}(T^{\Omega} - T_{\rm f}^{\rm SA}) \tag{27}$$

where the material parameter $T_{\rm s}^{\rm SA}$ is the temperature at which the martensite to austenite (SA) phase transformation starts. Following the procedure for the austenite to martensite model, the decrement in the volume fraction of martensite for the martensite to austenite phase change is

$$\Delta\xi_{\rm s}^{\Omega} = (\xi_{{\rm s}_n}^{\Omega})\left\{\frac{(\sigma^{\Omega} - \sigma_n^{\Omega}) - (T^{\Omega} - T_n^{\Omega})C^{\rm SA}}{(\sigma^{\Omega} - \tilde{\sigma}_{\rm f}^{\rm SA}) - [(\sigma^{\Omega} - \sigma_n^{\Omega}) - (T^{\Omega} - T_n^{\Omega})C^{\rm SA}]}\right\} \tag{28}$$

Eq. (28) is written in terms of the average strain in the composite as follows

$$\Delta\xi_{\rm s}^{\Omega} = (\xi_{{\rm s}_n}^{\Omega})\left[\frac{B^{\Omega} - (T^{\Omega} - T_n^{\Omega})C^{\rm SA}}{[C^{\Omega}{\rm A}^{\Omega}(Q^{\Omega}\gamma^{\Omega} + P^{\Omega}\bar{\varepsilon}) - \tilde{\sigma}_{\rm f}^{\rm SA}] - [(B^{\Omega}) - (T^{\Omega} - T_n^{\Omega})C^{\rm SA}]}\right] \tag{29}$$

where $B^{\Omega} = C^{\Omega}{\rm A}^{\Omega}[(Q^{\Omega}\gamma^{\Omega} + P^{\Omega}\bar{\varepsilon}) - (Q^{\Omega}\gamma_n^{\Omega} + P^{\Omega}\bar{\varepsilon}_n)]$

3.2. Constitutive equation for the matrix

The total strain in the isotropic matrix (M) is considered as the sum of elastic and hygrothermal strains.

$$\varepsilon^M = \varepsilon_{\rm el}^M + \alpha^M(T^M - T_0^M) + \beta_m^M(\chi^M - \chi_0^M) = \varepsilon_{\rm el}^M + \gamma^M \tag{30}$$

where $\beta_m^M, \chi^M, \chi_0^M$ and α^M are the moisture coefficient, the strain due to moisture, the reference moisture strain, the coefficient of thermal expansion and T_0^M is the reference temperature, respectively, for the matrix. The parameters ε^M and γ^M represent the total and inelastic hygrothermal strains in the matrix, respectively.

The stress–strain relationship for the matrix is defined as

$$\sigma^M = C^M\{\varepsilon_{\rm el}^M\} \tag{31}$$

where σ^M, C^M and $\varepsilon_{\rm el}^M$ are the stress, elastic modulus and elastic strain in the matrix, respectively.

4. Hygro-thermo-elastic constitutive equation for the SMA composite

The average properties of the SMA composite shown in Fig. 1 are derived using the Eshelby's classical stress consistency and iso-strain conditions under which the composite is assumed to be loaded. The fibres are usually of very small dimensions in the transverse directions as compared to the longitudinal direction. Hence, embedding such long fibres in a matrix will result in transversely isotropic elasticity on the plane normal to longitudinal loading direction of the composite.

The coupled total strain field ($\bar{\varepsilon}$) for the SMA composite is expressed as

$$\bar{\varepsilon} = \{\bar{\varepsilon}_{\rm el} + [\overline{P}]\gamma_p^{\Omega} + [\bar{\lambda}]\Delta T^{(\Omega+M)} + [\overline{\Lambda}]\gamma_m^M\} \tag{32}$$

where the parameters with (–) as superscripts represent the average values of strain tensors in the SMA composite and $\Delta T^{(\Omega+M)}$ is the temperature difference. The variables $\overline{P}, \bar{\lambda}$ and $\overline{\Lambda}$ are the phase transformation constant, thermo-elastic constant and hygro-thermo-elastic constant for the SMA composite, respectively. The variables $\gamma_{\rm p}^{\Omega}$ and $\gamma_{\rm m}^{M}$ are the inelastic strains due to phase transformations in the SMA wire and due to moisture expansion effects in the matrix, respectively, which are given by the following expressions

$$\gamma_{\rm p}^{\Omega} = \xi_{\rm s}^{\Omega}\varepsilon_{\rm L} \tag{33}$$

$$\gamma_{\rm m}^{M} = \beta_{\rm m}^{M}\{\chi^M - \chi_0^M\} \tag{34}$$

The coupled strain in the SMA composite lamina is rewritten in terms of composite strain tensors as

$$\bar{\varepsilon} = \{\bar{\varepsilon}_{\rm el} + \bar{\varepsilon}_{\rm p} + \bar{\varepsilon}_{\rm t} + \bar{\varepsilon}_{\rm m}\} \tag{35}$$

where the parameters $\bar{\varepsilon}_{\rm el}, \bar{\varepsilon}_{\rm p}, \bar{\varepsilon}_{\rm t}$ and $\bar{\varepsilon}_{\rm m}$ denote the average elastic strain, the average inelastic phase transformation strains, the average thermal strains and average hygric strains in the composite, respectively. The strain parameters are defined as

$$\bar{\varepsilon}_{\rm p} = [\overline{P}]\gamma_{\rm p}^{\Omega} \tag{36}$$

$$\bar{\varepsilon}_{\rm t} = [\bar{\lambda}]\Delta T^{(\Omega+M)} \tag{37}$$

$$\bar{\varepsilon}_{\rm m} = [\overline{\Lambda}]\gamma_{\rm m}^{M} \tag{38}$$

In the case of uniaxial loading (1–1 direction) of the composite cell, Eq. (35) is written as

$$\bar{\varepsilon}_{\rm el} = \bar{\varepsilon} - \overline{P}_{\rm p}\{\xi_{\rm s}^{\Omega}\varepsilon_{\rm L}\} - \bar{\lambda}_{\rm t}\{T - T_0\} - \overline{B}_{\rm m}\{\chi^M - \chi_0^M\} \tag{39}$$

where $\overline{P}_{\rm p}, \bar{\lambda}_{\rm t}$ and $\overline{B}_{\rm m}$ are the inelastic strain tensors for the composite cell.

Finally, the proposed constitutive equation for the SMA composite under hygro-thermo-elastic strain fields is expressed as

$$\bar{\sigma} = [\overline{C}]\{\bar{\varepsilon}_{\rm el}\} \tag{40}$$

where $\bar{\sigma}, \bar{\varepsilon}_{\rm el}$ and $\overline{C}$ are the average stress, the average elastic strain and homogenized elastic stiffness of the SMA composite, respectively.

4.1. Elastic stiffness of the SMA composite

In order to derive the elastic stiffness of the SMA composite, only elastic deformations of the fibre and matrix are considered while all other inelastic deformations are set equal to zero. The constitutive equation of the matrix, Eq. (31), is rewritten using average elastic strain equation for the composite as

$$\vartheta^M\sigma^M = C^M\{\bar{\varepsilon}_{\rm el} - \varepsilon_{\rm el}^{\Omega}\vartheta^{\Omega}\} \tag{41}$$

The average stress equation of the composite, is rewritten as

$$\vartheta^M\sigma^M = \overline{C}\{\bar{\varepsilon}_{\rm el}\} - C^{\Omega}\{\varepsilon_{\rm el}^{\Omega}\vartheta^{\Omega}\} \tag{42}$$

Substituting Eq. (16) into Eq. (42) results in

$$\vartheta^M\sigma^M = \overline{C}\{\bar{\varepsilon}_{\rm el}\} - C^{\Omega}\{[\gamma^{\Omega} + A^{\Omega}(P\bar{\varepsilon}_{\rm el} + Q\gamma^{\Omega})]\vartheta^{\Omega}\} \tag{43}$$

By equating (41) and (43), the following expression is obtained

$$\overline{C}\{\bar{\varepsilon}_{\rm el}\} = C^M\{\bar{\varepsilon}_{\rm el} - \varepsilon_{\rm el}^{\Omega}\vartheta^{\Omega}\} + C^{\Omega}\{[\gamma^{\Omega} + A^{\Omega}(P\bar{\varepsilon}_{\rm el} + Q\gamma^{\Omega})]\vartheta^{\Omega}\} \tag{44}$$

The above equation is simplified and the overall elastic moduli of the composite is derived as

$$\overline{C} = C^M\{I - [(C^M - C^{\Omega})^{-1}C^M - S^{\Omega}]^{-1}\vartheta^{\Omega}\} \tag{45}$$

where, I is the fourth order identity tensor and $\overline{C}$ is the overall elastic moduli of the SMA composite.

4.2. Effect of inelastic strain due to phase transformations in the SMA inclusion

The average strain in the composite $(\overline{P}_{\mathrm{p}})$ due to the eigen strain in the inclusion $(\pi_{\mathrm{p}}^{\Omega})$ is computed by setting the external tractions or the applied stress to the composite as zero.

$$0 = \sigma_{\mathrm{p}}^{\Omega}\vartheta^{\Omega} + \sigma^{M}\vartheta^{M} \tag{46}$$

where $\sigma_{\mathrm{p}}^{\Omega}$ corresponds to the eigen stress induced due to the unit inelastic eigen strain in the inclusion.

Following the procedure given in reference (Marfia and Sacco, 2005), in the formulas (1)–(11), it is set as $\bar{\varepsilon} = \overline{P}_{\mathrm{p}}, \varepsilon^{\Omega} = \varepsilon_{\mathrm{p}}^{\Omega}$ and $\gamma^{\Omega} = \pi_{\mathrm{p}}^{\Omega}$, which results in average elastic strain in the inclusion as $\varepsilon_{\mathrm{p}}^{\Omega} - \pi_{\mathrm{p}}^{\Omega}$. Therefore, the stress in the inclusion and matrix are written in terms of eigen strain in the inclusion as

$$\sigma_{\mathrm{p}}^{\Omega} = C^{\Omega}\{\varepsilon_{\mathrm{p}}^{\Omega} - \pi_{\mathrm{p}}^{\Omega}\} \tag{47}$$

$$\vartheta^{M}\sigma^{M} = C^{M}\{\overline{P}_{\mathrm{p}} - \varepsilon_{\mathrm{p}}^{\Omega}\vartheta^{\Omega}\} \tag{48}$$

where $\overline{P}_{\mathrm{p}}$ is the average inelastic strain in the composite due to the total inelastic strain in the inclusion.

Substituting Eqs. (47) and (48) into Eq. (46) yields

$$0 = -\vartheta^{\Omega}\{C^{\Omega}\pi_{\mathrm{p}}^{\Omega}\} + C^{M}\{\overline{P}_{\mathrm{p}}\} + \vartheta^{\Omega}[C^{\Omega} - C^{M}](\varepsilon_{\mathrm{p}}^{\Omega}) \tag{49}$$

Similar to Eq. (8), the total inelastic strain in the inclusion due to phase transformations is written as

$$\varepsilon_{\mathrm{p}}^{\Omega} = \pi_{\mathrm{p}}^{\Omega} + A^{\Omega}\{P^{\Omega}\overline{P}_{\mathrm{p}} + Q^{\Omega}\pi_{\mathrm{p}}^{\Omega}\} \tag{50}$$

Substituting Eq. (50) into Eq. (49) the overall inelastic strain tensor of SMA composite due to phase transformation in the SMA inclusion is

$$\overline{P}_{\mathrm{p}} = \vartheta^{\Omega}[\{I - \vartheta^{\Omega}P^{\Omega}\}]^{-1}[\{I + Q\}]\{\pi_{\mathrm{p}}^{\Omega}\} \tag{51}$$

This expression is the redefined phase transformation strain tensor ($\overline{P}_{\mathrm{p}} \neq \overline{P}$, where $\overline{P}$ is given in the reference Marfia and Sacco, 2005) for the SMA composite, which gives the phase transformation constant for the SMA composite under unit inelastic strain as

$$\overline{P} = \vartheta^{\Omega}[\{I - \vartheta^{\Omega}P^{\Omega}\}]^{-1}[\{I + Q\}] \tag{51a}$$

4.3. Effect of inelastic strain due to moisture content in the matrix

Following the procedure listed above, the expression for inelastic strain in the composite due to the total inelastic hygric strain induced in the matrix is derived. The total strain in the composite due to the moisture content in the matrix is set as $\overline{B}_{\mathrm{m}}$.

The total average stress in the composite due to hygric strain in the matrix is

$$0 = \sigma^{\Omega}\vartheta^{\Omega} + \sigma_{\mathrm{m}}^{M}\vartheta^{M} \tag{52}$$

The constitutive relations for the fibre and matrix are given as

$$\sigma^{\Omega} = C^{\Omega}\{\varepsilon_{\mathrm{m}}^{\Omega} - \pi_{\mathrm{m}}^{\Omega}\} \tag{53}$$

$$\sigma^{M} = C^{M}\{\varepsilon_{\mathrm{m}}^{M} - \pi_{\mathrm{m}}^{M}\} \tag{54}$$

where $\pi_{\mathrm{m}}^{\Omega}$ and π_{m}^{M} are the inelastic strains due to moisture in the inclusion and matrix, respectively. However, the SMA wire is not induced by hygric strains and as a result the inelastic strain $(\pi_{\mathrm{m}}^{\Omega})$ is neglected. Substituting Eqs. (53) and (54) into Eq. (52) results in

$$0 = \vartheta^{\Omega}C^{\Omega}\{\varepsilon_{\mathrm{m}}^{\Omega}\} + \vartheta^{M}C^{M}\{\varepsilon_{\mathrm{m}}^{M} - \pi_{\mathrm{m}}^{M}\} \tag{55}$$

Since the SMA wire is embedded in the matrix, the relative strain in the SMA wire due to hygric strain in the composite $(\overline{B}_{\mathrm{m}})$ is

$$\varepsilon_{\mathrm{m}}^{\Omega} = A^{\Omega}P^{\Omega}\{\overline{B}_{\mathrm{m}}\} \tag{56}$$

Incorporating the total strain in the inclusion into Eq. (55) results in

$$0 = \vartheta^{\Omega}[C^{\Omega}A^{\Omega}P^{\Omega}]\{\overline{B}_{\mathrm{m}}\} + \vartheta^{M}[C^{M}]\{\varepsilon_{\mathrm{m}}^{M} - \pi_{\mathrm{m}}^{M}\} \tag{57}$$

Using the average strain equation for the composite, the strain in the matrix due to the moisture is written as follows

$$\vartheta^{M}\varepsilon_{\mathrm{m}}^{M} = \overline{B}_{\mathrm{m}} - \vartheta^{\Omega}\varepsilon_{\mathrm{m}}^{\Omega} \tag{58}$$

Eq. (56) is substituted into Eq. (58) and the total strain in the matrix is obtained as,

$$\vartheta^{M}\varepsilon_{\mathrm{m}}^{M} = \overline{B}_{\mathrm{m}} - \vartheta^{\Omega}A^{\Omega}P^{\Omega}\overline{B}_{\mathrm{m}} \tag{59}$$

Substituting the above expression into Eq. (55) results in

$$0 = \{\vartheta^{\Omega}[C^{\Omega}A^{\Omega}P^{\Omega}] + [C^{M} - \vartheta^{\Omega}C^{M}A^{\Omega}P^{\Omega}]\}\{\overline{B}_{\mathrm{m}}\} - [\vartheta^{M}C^{M}]\{\pi_{\mathrm{m}}^{M}\} \tag{60}$$

Using the above equation, the second order strain tensor $\overline{B}_{\mathrm{m}}$ in the composite due to the unit hygric change effect in the matrix is

$$\overline{B}_{\mathrm{m}} = [\{I - \vartheta^{\Omega}P^{\Omega}\}]^{-1}\vartheta^{M}\{\pi_{\mathrm{m}}^{M}\} \tag{61}$$

This expression is the hygric strain tensor, which gives the hygric tensor for the SMA composite in the following form

$$\overline{\Lambda} = [\{I - \vartheta^{\Omega}P^{\Omega}\}]^{-1}\vartheta^{M}\{\beta_{m}^{M}\} \tag{61a}$$

4.4. Effect of thermal deformation in the SMA composite

Based on the above simplifications (Sections 4.2 and 4.3), the total thermal deformation due to a unit temperature change in the composite is evaluated in terms of the difference in coefficient of thermal expansions of the matrix and inclusion. The average strain in the composite due to the eigen strain in the inclusion is computed by setting the external tractions or the applied stress to the composite as zero.

$$0 = \sigma_{\mathrm{t}}^{\Omega}\vartheta^{\Omega} + \sigma_{\mathrm{t}}^{M}\vartheta^{M} \tag{62}$$

Due to the fact that both the SMA wire and matrix undergo temperature induced strains $\varepsilon_{\mathrm{t}}^{\Omega}$ and $\varepsilon_{\mathrm{t}}^{M}$, respectively, the constitutive relations for SMA wire and matrix are expressed as

$$\sigma^{\Omega} = C^{\Omega}\{\varepsilon_{\mathrm{t}}^{\Omega} - \pi_{\mathrm{t}}^{\Omega}\} \tag{63}$$

$$\sigma^{M} = C^{M}\{\varepsilon_{\mathrm{t}}^{M} - \pi_{\mathrm{t}}^{M}\} \tag{64}$$

where $\pi_{\mathrm{t}}^{\Omega}$ and π_{t}^{M} are unit thermal strain tensors in the SMA wire and matrix, respectively.

Substituting the above relations into Eq. (62) and using the average strain equation of the composite subjected to thermal strain ($\bar{\lambda}_{\mathrm{t}}$), yields

$$0 = [I - \vartheta^{\Omega}P^{\Omega}]\bar{\lambda}_{\mathrm{t}} - [\vartheta^{\Omega}Q^{\Omega}\pi_{\mathrm{t}}^{\Omega} + \pi_{\mathrm{t}}^{M}] - \vartheta^{\Omega}\{\pi_{\mathrm{t}}^{\Omega} - \pi_{\mathrm{t}}^{M}\} \tag{65}$$

Finally, the equation for the thermal strain tensor for a unit temperature change is

$$\bar{\lambda}_{\mathrm{t}} = [I - \vartheta^{\Omega}P^{\Omega}]^{-1}[\{\vartheta^{\Omega}Q^{\Omega}\pi_{\mathrm{t}}^{\Omega} + \pi_{\mathrm{t}}^{M}\} + \vartheta^{\Omega}\{\pi_{\mathrm{t}}^{\Omega} - \pi_{\mathrm{t}}^{M}\}] \tag{66}$$

where I is the fourth order identity tensor and ($\bar{\lambda}_{\mathrm{t}} \neq \bar{\iota}$, where $\bar{\iota}$ is the thermal strain tensor for SMA composite cell given in the reference Marfia and Sacco, 2005).

This expression (66) gives the thermo-elastic constant for the SMA composite, which is written as

$$\bar{\lambda} = [I - \vartheta^{\Omega}P^{\Omega}]^{-1}[\{\vartheta^{\Omega}(I + Q^{\Omega})\alpha^{\Omega}\} - \vartheta^{M}I\alpha^{M}] \tag{66a}$$

Finally, the thermo-elastic coefficients ($\bar{\alpha}$) and hygro-elastic coefficients ($\bar{\beta}$) of the SMA composite can be evaluated using Eqs. (61a) and (66a), respectively.

5. Evaluation of overall SMA composite (unidirectionally reinforced) lamina properties

The overall stiffness of the SMA composite is evaluated by using Eshelby's tensor (Mura, 1982) derived for long cylindrical fibres of isotropic behavior. Eshelby's equivalent inclusion method is used to characterize and develop a unidirectionally SMA wire reinforced composite lamina represented in Fig. 2. The assumption of such a lamina is valid because the SMA wires have high modulus than the matrix and therefore the lamina cannot withstand high stresses in any direction other than that of wires. Another important point in case of unidirectional SMA wire lamina is that the phase transformations as supposed to occur along the length of the SMA wire, which in turn can be considered as the loading and actuation direction for the SMA composite.

5.1. On-axis properties of the SMA composite lamina

For a unidirectionally reinforced lamina in the 1–2 plane (Fig. 2), a plane stress state is defined by the following loading conditions (Daniel and Ishai, 1994)

$$\bar{\sigma}_{33} = 0; \quad \bar{\tau}_{23} = 0 \quad \text{and} \quad \bar{\tau}_{13} = 0 \tag{67}$$

Applying the above conditions for the composite cell will result in the following relations for stress state

$$\bar{\sigma}_{11} \neq 0; \quad \bar{\sigma}_{22} \neq 0 \quad \text{and} \quad \bar{\tau}_{12} \neq 0 \tag{68}$$

The on-axis hygro-thermo-elastic stress–strain relations are given in terms of the reduced stiffness coefficients ($\overline{Q}_{ij}$) in material coordinates by the following constitutive equation (Daniel and Ishai, 1994)

$$\begin{Bmatrix} \bar{\sigma}_{11} \\ \sigma_{22} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & 0 \\ \overline{Q}_{12} & \overline{Q}_{22} & 0 \\ 0 & 0 & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} \bar{e}_{11} \\ \bar{e}_{22} \\ \bar{\varphi}_{12} \end{Bmatrix} \tag{69}$$

where $\bar{e}$ and $\bar{\varphi}$ are the elastic strains and shear strain in material coordinates, respectively.

The coefficients of reduced stiffnesses for the lamina after static condensation are given by the expressions

$$\overline{Q}_{11} = \overline{C}_{11} - \frac{\overline{C}_{13}\overline{C}_{13}}{\overline{C}_{33}}; \quad \overline{Q}_{12} = \overline{C}_{12} - \frac{\overline{C}_{13}\overline{C}_{23}}{\overline{C}_{33}}; \quad \overline{Q}_{22} = \overline{C}_{22} - \frac{\overline{C}_{23}\overline{C}_{23}}{\overline{C}_{33}}; \quad \overline{Q}_{66} = \overline{C}_{66} \tag{70}$$

From the concept of Eshelby method it is clear that the strain tensors ($\overline{P}_{\rm p}$, $\bar{\lambda}_{\rm t}$ and $\overline{B}_{\rm m}$) as well as the phase transformation, thermo-elastic and hygro-thermo-elastic constants ($\overline{P}$, $\bar{\lambda}$ and $\overline{\Lambda}$) for fibre reinforced composites depend upon the components of Eshelby tensor (S^{Ω}). The static condensation ($*$) is also carried out on the coefficients of these constants and the following form of elastic strain ($\bar{e}$) for unidirectional loading of the composite lamina under plane stress condition is written

$$\bar{e}_{11} = \bar{\varepsilon}_{11} - [\overline{P}^{*}_{11}]\{\xi^{\Omega}_{\rm s}\varepsilon_{\rm L}\} - [\bar{\lambda}^{*}_{11}]\{T - T_0\} - [\overline{\Lambda}^{*}_{11}]\{\chi^{M} - \chi^{M}_{0}\} \tag{71}$$

The effective compliance of the composite is exactly the inverse of stiffness matrix given by the relation

$$[\overline{U}] = [\overline{C}]^{-1} \tag{72}$$

Finally, for unidirectional loading of the SMA composite Eq. (69) is expressed as

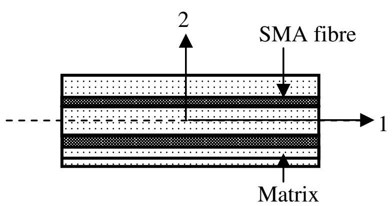

Fig. 2. SMA reinforced composite lamina.

$$\{\bar{\sigma}_{11}\} = [\overline{Q}_{11}^*]\{\bar{e}_{11}\} \tag{73}$$

where $\overline{Q}_{11}^*$ is the reduced stiffness in the direction of loading and is given as

$$[\overline{Q}_{11}^*] = \left[\overline{Q}_{11} - \frac{\overline{Q}_{12}\overline{Q}_{12}}{\overline{Q}_{22}}\right] \tag{74}$$

5.2. Off-axis properties of the SMA composite

The off-axis properties of SMA composite are important when the lamina is loaded along the arbitrary axes X and Y. Therefore the off-axis stress–strain relationship in the X–Y–Z direction (Daniel and Ishai, 1994) is

$$\begin{Bmatrix} \bar{\sigma}_{xx} \\ \bar{\sigma}_{yy} \\ \bar{\tau}_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{xx}^* & Q_{xy}^* & 2Q_{xs}^* \\ Q_{xy}^* & Q_{yy}^* & 2Q_{ys}^* \\ Q_{xy}^* & Q_{xy}^* & 2Q_{ss}^* \end{bmatrix} \begin{Bmatrix} \bar{e}_{xx} \\ \bar{e}_{yy} \\ 0.5\varphi_{xy} \end{Bmatrix} \tag{75}$$

where $\{\bar{\sigma}\}_{xyz}, \{\bar{e}\}_{xyz}$ and $[Q^*]$ are the average stresses, the average strains and reduced transformed coefficients of the stiffness matrix in the X–Y–Z coordinate system.

The reduced transformed coefficients are given by the following relations

$$Q^* = [T_r]^T[\overline{Q}][T_r] \tag{76a}$$
$$\overline{P}^* = [T_r]^T[\overline{P}][T_r] \tag{76b}$$
$$\bar{\lambda}^* = [T_r]^T[\bar{\lambda}][T_r] \tag{76c}$$
$$\overline{\Lambda}^* = [T_r]^T[\overline{\Lambda}][T_r] \tag{76d}$$

where T_r is the well-known standard two-dimensional transformation matrix.

The transformed coefficients of the average stiffness (Q^*) and average compliance coefficients (U^*) (Daniel and Ishai, 1994) for transversely isotropic plane stress conditions are given below.

$$\begin{aligned}
Q_{xx}^* &= \overline{Q}_{11}m^4 + \overline{Q}_{22}n^4 + \overline{Q}_{12}2m^2n^2 + \overline{Q}_{66}4m^2n^2 \\
Q_{yy}^* &= \overline{Q}_{11}n^4 + \overline{Q}_{22}m^4 + \overline{Q}_{12}2m^2n^2 + \overline{Q}_{66}4m^2n^2 \\
Q_{xy}^* &= \overline{Q}_{11}m^2n^2 + \overline{Q}_{22}m^2n^2 + \overline{Q}_{12}(m^4 + n^4) - \overline{Q}_{66}4m^2n^2 \\
Q_{ss}^* &= \overline{Q}_{11}m^2n^2 + \overline{Q}_{22}m^2n^2 - \overline{Q}_{12}2m^2n^2 + \overline{Q}_{66}(m^2 - n^2)^2 \\
Q_{xs}^* &= \overline{Q}_{11}m^3n - \overline{Q}_{22}mn^3 + \overline{Q}_{12}(mn^3 - m^3n) + \overline{Q}_{66}2(mn^3 - m^3n) \\
Q_{ys}^* &= \overline{Q}_{11}mn^3 - \overline{Q}_{22}m^3n + \overline{Q}_{12}(m^3n - mn^3) + \overline{Q}_{66}2(m^3n - mn^3)
\end{aligned} \tag{77}$$

$$\begin{aligned}
U_{xx}^* &= \overline{U}_{11}m^4 + \overline{U}_{22}n^4 + \overline{U}_{12}2m^2n^2 + \overline{U}_{66}m^2n^2 \\
U_{yy}^* &= \overline{U}_{11}n^4 + \overline{U}_{22}m^4 + \overline{U}_{12}2m^2n^2 + \overline{U}_{66}m^2n^2 \\
U_{xy}^* &= \overline{U}_{11}m^2n^2 + \overline{U}_{22}m^2n^2 + \overline{U}_{12}(m^4 + n^4) - \overline{U}_{66}m^2n^2 \\
U_{ss}^* &= \overline{U}_{11}4m^2n^2 + \overline{U}_{22}4m^2n^2 - \overline{U}_{12}8m^2n^2 + \overline{U}_{66}(m^2 - n^2)^2 \\
U_{xs}^* &= \overline{U}_{11}2m^3n - \overline{U}_{22}2mn^3 + \overline{U}_{12}2(mn^3 - m^3n) + \overline{U}_{66}(mn^3 - m^3n) \\
U_{ys}^* &= \overline{U}_{11}2mn^3 - \overline{U}_{22}2m^3n + \overline{U}_{12}2(m^3n - mn^3) + \overline{U}_{66}(m^3n - mn^3)
\end{aligned} \tag{78}$$

where $m = \text{cosine}\,\theta$ and $n = \text{sine}\,\theta$

The off-axis equations for transformed reduced coefficients of $\overline{P}^*, \overline{\Lambda}^*$ and $\bar{\lambda}^*$ can be expressed similar to Eq. (77). The off-axis stress strain relation for unidirectional loading of the SMA lamina is

$$\bar{\sigma}_{xx} = [\overline{Q}_{11}^*]\{\bar{e}_{xx}\} \tag{79}$$

where $\overline{Q}_{11}^*$ is the condensed reduced transformed coefficient and $\bar{e}_{xx}$ is the off-axis elastic strain.

The thermo-elastic ($\bar{\alpha}_{xyz}$) and hygro-thermo-elastic coefficients ($\bar{\beta}_{xyz}$) of SMA composite lamina are obtained by using the Eqs. (76c) and (76d). These coefficients are functions of Eshelby tensor (S^{Ω}) and are given below using Eqs. (61a) and (66a), respectively.

$$\bar{\alpha}_{xx} = \bar{\lambda}_{11}m^2 + \bar{\lambda}_{22}n^2, \bar{\alpha}_{yy} = \bar{\lambda}_{11}n^2 + \bar{\lambda}_{22}m^2, \bar{\alpha}_{xy} = mn\bar{\lambda}_{66} \tag{80}$$

$$\bar{\beta}_{xx} = \overline{\Lambda}_{11}m^2 + \overline{\Lambda}_{22}n^2, \bar{\beta}_{yy} = \overline{\Lambda}_{11}n^2 + \overline{\Lambda}_{22}m^2, \bar{\beta}_{xy} = mn\overline{\Lambda}_{66} \tag{81}$$

The shear coefficients in Eqs. (80) and (81) for unidirectional lamina with principal material axis are defined as

$$\bar{\alpha}_{66} = 2(\bar{\alpha}_{11} - \bar{\alpha}_{22})mn; \quad \bar{\beta}_{66} = 2(\bar{\beta}_{11} - \bar{\beta}_{22})mn \tag{82}$$

By substituting the on-axis SMA composite elastic properties in the compliance transformation equations, the following transformation relations for the engineering constants of the SMA composite lamina are obtained.

$$\begin{gathered}
\frac{1}{\overline{E}_{xx}} = \frac{m^2}{\overline{E}_{11}}(m^2 - n^2\bar{\upsilon}_{12}) + \frac{n^2}{\overline{E}_{22}}(n^2 - m^2\bar{\upsilon}_{21}) + \frac{m^2n^2}{\overline{G}_{12}}; \quad \frac{1}{\overline{E}_{yy}} = \frac{n^2}{\overline{E}_{11}}(n^2 - m^2\bar{\upsilon}_{12}) + \frac{m^2}{\overline{E}_{22}}(m^2 - n^2\bar{\upsilon}_{21}) + \frac{m^2n^2}{\overline{G}_{12}} \\
\frac{\upsilon_{xy}}{\overline{E}_{xx}} = \frac{\upsilon_{yx}}{\overline{E}_{yy}} = \frac{m^2}{\overline{E}_{11}}(m^2\bar{\upsilon}_{12} - n^2) + \frac{n^2}{\overline{E}_{22}}(n^2\bar{\upsilon}_{21} - m^2) + \frac{m^2n^2}{\overline{G}_{12}}; \quad \frac{1}{\overline{G}_{xy}} = \frac{4m^2n^2}{\overline{E}_{11}}(1 + \bar{\upsilon}_{12}) + \frac{4m^2n^2}{\overline{E}_{22}}(1 + \bar{\upsilon}_{21}) + \frac{(m^2 - n^2)^2}{\overline{G}_{12}}
\end{gathered} \tag{83}$$

where the engineering constants are given by the following relations

$$\overline{E}_{11} = \overline{Q}_{11}(1 - \upsilon_{12}\upsilon_{21}); \quad \bar{\upsilon}_{12} = \frac{\overline{Q}_{12}(1 - \upsilon_{12}\upsilon_{21})}{\overline{E}_{22}}; \quad \frac{\bar{\upsilon}_{12}}{\overline{E}_{11}} = \frac{\bar{\upsilon}_{21}}{\overline{E}_{22}}; \quad \overline{E}_{22} = \overline{Q}_{22}(1 - \upsilon_{12}\upsilon_{21}); \quad \overline{G}_{12} = \overline{Q}_{66} \tag{84}$$

6. Results, discussions and observations

In this study, the pseudoelastic response of SMA wire reinforced composite cell as well as lamina subjected to hygro-thermo-elastic strain field is investigated through analytical procedure based on Eshelby's equivalent inclusion method. The components of Eshelby tensor for long cylindrical fibre (SMA wire) are given in Table 1 and the material properties for the matrix and SMA wire are given in Table 2. The properties of matrix used for hygro-thermo-elastic computations are listed in Table 3.

6.1. Analytical simulations of the pseudoelastic behavior of SMA composite

This section presents some analytical simulations developed in order to illustrate the capability of the analytical procedure to capture the general thermo-mechanical behavior of SMA composite. Basically, analytical results related to pseudoelasticity under uniaxial tensile loading of the composite due to complete phase transformations are addressed. Moreover, in order to perform the most basic test on the analytical procedure and in order to present a basis for the subsequent applications of SMA composites, uniaxial tensile computations illustrating the pseudoelastic response of SMAs are performed at temperature of 60 °C under hygro-thermo-elastic strain fields for the SMA composite cell as well as the lamina. In the present computation it is assumed that the SMA composite is not constrained against any deformation and as a result the mechanical stresses are zero in the initial state. Also, due to the fact that SMA wire has high Young's modulus than the matrix material in the loading direction, the moisture expansion strain as prestrain is neglected and free moisture expan-

Table 1
Eshelby tensor for long cylindrical fibre (Mura, 1982)

$S^{\Omega}_{1111} = S^{\Omega}_{1122} = S^{\Omega}_{1133} = 0$,	$S^{\Omega}_{2233} = S^{\Omega}_{3322} = \left(\frac{1-4\upsilon^M}{8(1-\upsilon^M)}\right)$,	$S^{\Omega}_{2211} = S^{\Omega}_{3311} = \left(\frac{\upsilon^M}{2(1-\upsilon^M)}\right)$
$S^{\Omega}_{2323} = \left(\frac{3-4\upsilon^M}{8(1-\upsilon^M)}\right)$,	$S^{\Omega}_{1313} = S^{\Omega}_{1212} = \left(\frac{1}{4}\right)$,	$S^{\Omega}_{3333} = S^{\Omega}_{2222} = \left(\frac{5-4\upsilon^M}{8(1-\upsilon^M)}\right)$

Table 2
Material properties of epoxy matrix (M) and SMA fibre (Ω) (Marfia and Sacco, 2005)

$C^M = 3600$ MPa,	$\upsilon^M = 0.305$,	$\alpha^M = 0/°\text{C}$,	$C^\Omega = 70,000$ MPa,	$\varepsilon_L = 7\%\upsilon^\Omega = 0.33$	
$G^\Omega = 46,550$ MPa,	$\alpha^\Omega = 10 \times 10^{-6}/°\text{C}$,	$\sigma_S^{AS} = 140$ MPa,	$\sigma_f^{AS} = 200$ MPa,	$T_f^{AM} = 5\,°\text{C}$	
$T_s^{AM} = 10\,°\text{C}$,	$T_s^{SA} = 30\,°\text{C}$,	$T_f^{SA} = 31\,°\text{C}$,	$C^{AS} = 8$ MPa/°C,	$C^{SA} = 8$ MPa/°C	

Table 3
Elastic, thermal and hygric properties of epoxy matrix (M) (Daniel and Ishai, 1994)

$C^M = 3450$ MPa,	$\upsilon^M = 0.35$,	$G^M = 1280$ MPa,	$\alpha^M = 64.3 \times 10^{-6}/°\text{C}$,	$\beta_m^M = 0.38$

sion strain is considered for the analytical computations. Therefore, the initial total strain is equal to free moisture expansion strains ($\bar{\varepsilon} = \bar{\varepsilon}_m = \overline{\Lambda}\gamma_m^M$) in the composite. The pseudoelastic behavior of the SMA wire as function of the average composite strain is shown in Fig. 3. The corresponding variation in martensite fraction for complete phase transformation under the external loading of composite is also presented in Fig. 4.

In this paper, two inelastic strain tensors, namely; $\overline{P}_p$ due to phase transformations in the SMA wire and $\bar{\lambda}_t$ due to temperature change in the wire and matrix are modified while the inelastic strain tensor $\overline{B}_m$ due to moisture expansion in the matrix is newly derived. Using these strain tensors, the phase transformation constant $\overline{P}$, the thermo-elastic constant $\bar{\lambda}$ and hygro-thermo-elastic constant $\overline{\Lambda}$ are also proposed so as to characterize the SMA lamina for the purpose of practical applications. As a result, in order to validate the present analytical procedure with the proposed modifications, the results for the SMA composite unit cell subjected to thermo-elastic strain field are compared with the results published by Marfia and Sacco (2005). Further, the stress–strain response for the SMA composite cell under hygro-thermo-elastic strain field for different SMA volume fractions is presented. The elastic stiffness of the SMA composite is evaluated using Eq. (45) and the values are listed in Table 4. The validation for pseudoelastic behavior of SMA composite cell is shown in Figs. 5 and 6 for low and high SMA volume fractions. The analytical results are also validated in terms of the average transformation stresses at the start and end of austenite to martensite phase transformations, which is shown in Table 5. The results are computed for four different SMA volume fractions based on dilute distribution theory. The average transformations stresses for the reverse phase transformation from martensite to austenite are presented for comparison in Table 6. From the above results it is observed that the present analytical procedure is able to investigate the pseudoelastic behavior of SMA composite cell with small variations in the overall hysteresis, which is due to the proposed modifications for the average strain tensors ($\overline{P}_p$ and $\bar{\lambda}_t$). The analytical procedure is now extended to evaluate the stress–strain hysteresis behavior of SMA composite cell under the influence of moisture expansion strains in the matrix. The inelastic strain tensor ($\overline{B}_m$) derived in Eq.

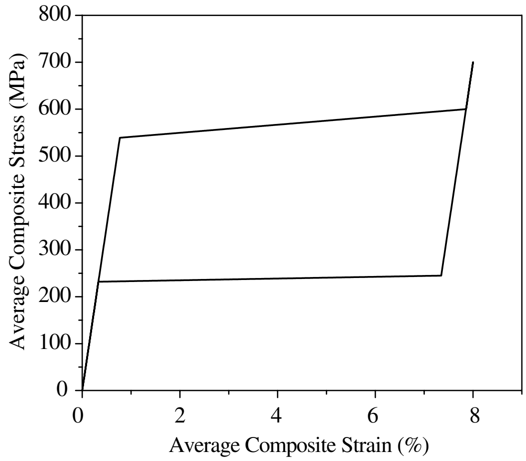

Fig. 3. Pseudoelastic behavior of SMA wire.

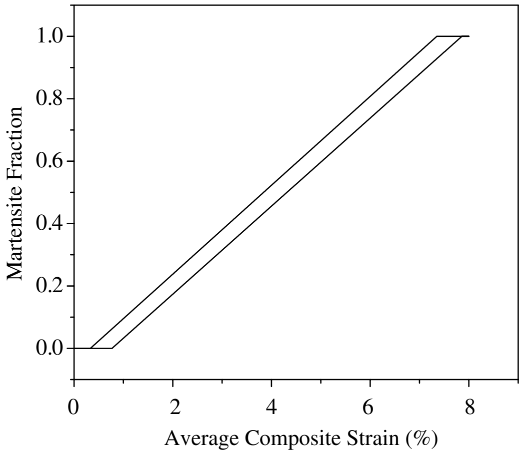

Fig. 4. Martensite fraction versus strain.

Table 4
Overall elastic stiffness [MPa] of SMA composite for $\vartheta^{\Omega} = 0.036$

	$\overline{C}_{11}$	$\overline{C}_{22} = \overline{C}_{33}$	$\overline{C}_{44}$	$\overline{C}_{55} = \overline{C}_{66}$	$\overline{C}_{12} = \overline{C}_{13}$	$\overline{C}_{23} = \overline{C}_{32}$
Long cylindrical	7378.89	5153.31	1511.56	1541.95	2267.29	2251.93
SMA wire inclusion	(7378.89)	(5153.32)	(1511.56)	(1541.94)	(2267.93)	(2251.93)

The values in the bracket are from Marfia and Sacco (2005).

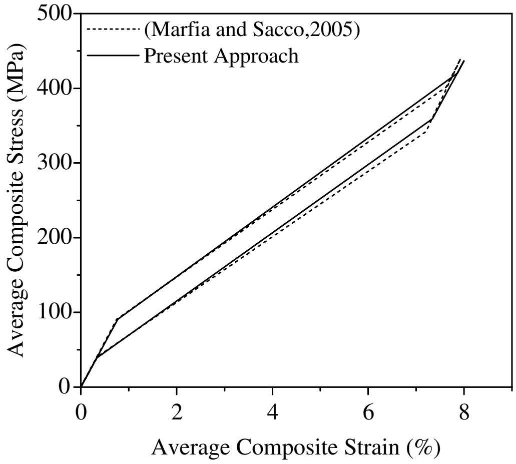

Fig. 5. SMA-composite cell behavior under thermo-elastic strain field and validation for $\vartheta^{\Omega} = 0.05$ and $T = 60$ °C.

(61) for the composite is used for the computation. The elastic strain is computed using Eq. (39). The average transformation stresses of the SMA composite for austenite to martensite and martensite to austenite phase transformations under the coupled hygro-thermo-elastic strain field are computed using Eq. (40). The analytical results for austenite to martensite phase change are presented in Table 7 and the results for the reverse phase transformations are given in Table 8. The results for thermo-elastic and hygro-thermo-elastic stress–strain behavior for the composite are plotted in Figs. 7 and 8, respectively. In Fig. 7, the stress–strain behavior is presented for low and high moisture strains in matrix with SMA volume fraction of 0.05 while in Fig. 8 the volume fraction is slightly higher (0.10). The important observation from comparison of these Figs. 6–9 is that the effect of hygric strains in matrix greatly influences the average transformation stresses of the overall com-

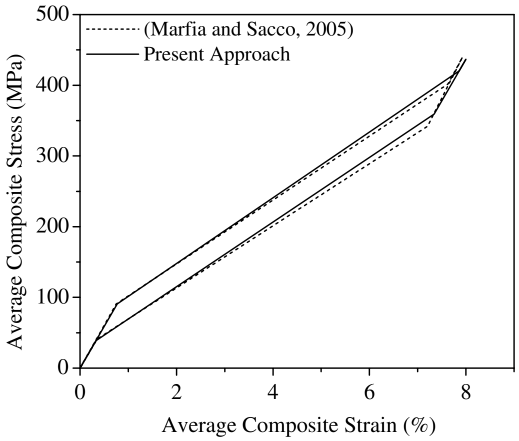

Fig. 6. SMA-composite cell behavior under thermo-elastic strain field and validation for $\vartheta^{\Omega} = 0.10$ and $T = 60$ °C.

Table 5
Average transformation stresses [MPa] of the SMA composite cell for austenite to martensite transformation: thermo-elastic strain field

	ϑ^{Ω}	0.01	0.036	0.05	0.10
Present approach	$\bar{\sigma}_{s}^{AS}$	43.20(43.10)	56.92(55.41)	64.31(64.28)	90.69(89.93)
Present approach	$\bar{\sigma}_{f}^{AS}$	389.78(381.14)	398.57(386.60)	403.30(390.52)	420.20(401.88)

The values in the bracket are from Marfia and Sacco (2005).

Table 6
Average transformation stresses [MPa] of SMA composite cell for martensite to austenite transformation: thermo-elastic strain field

Present approach	ϑ^{Ω}	0.01	0.036	0.05	0.10
Transformation start stresses	$\bar{\sigma}_{s}^{SA}$	360.82	360.41	360.19	359.40
Transformation finish stresses	$\bar{\sigma}_{f}^{SA}$	18.56	24.45	27.63	38.96

Table 7
Average HTE transformation stresses [MPa] of the SMA composite cell for austenite to martensite transformation: hygro-thermo-elastic strain field

Present approach		ϑ^{Ω}	0.01	10.036	0.05	0.10
Transformation start stresses	$\bar{\sigma}_{s}^{AS}$	$\chi^{M} = 0.001$	46.29	60.66	68.27	95.18
		$\chi^{M} = 0.005$	39.37	56.03	64.34	92.84
Transformation finish stresses	$\bar{\sigma}_{f}^{AS}$	$\chi^{M} = 0.001$	436.80	445.99	450.81	467.77
		$\chi^{M} = 0.005$	429.88	441.36	446.94	465.43

Table 8
Average HTE transformation stresses [MPa] of the SMA composite cell for martensite to austenite transformation: hygro-thermo-elastic strain field

Present approach		ϑ^{Ω}	0.01	0.036	0.05	0.10
Transformation start stresses	$\bar{\sigma}_{s}^{SA}$	$\chi^{M} = 0.001$	404.61	404.55	404.40	403.58
		$\chi^{M} = 0.005$	397.69	399.92	400.53	401.24
Transformation finish stresses	$\bar{\sigma}_{f}^{SA}$	$\chi^{M} = 0.001$	18.91	25.40	28.79	40.56
		$\chi^{M} = 0.005$	11.98	20.78	24.90	38.22

posite, which will in turn vary the hysteresis loops of the composite. This variation will directly affect the potential energy as well as the damping capacity of the overall composite for practical applications.

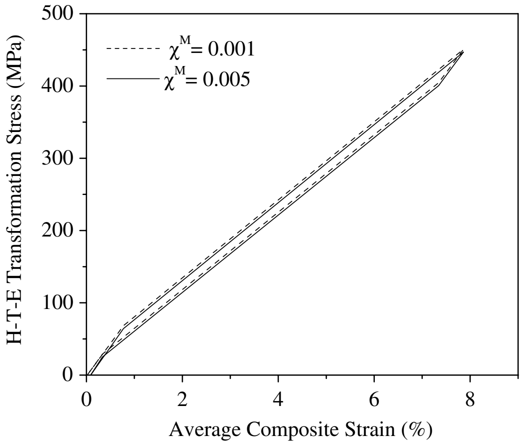

Fig. 7. SMA-composite cell behavior under hygro-thermo-elastic strain field for $\vartheta^{\Omega} = 0.05$, $\beta_{\mathrm{m}}^{M} = 0.38$, $\chi^{M} = 0.001$ and $\chi^{M} = 0.005$.

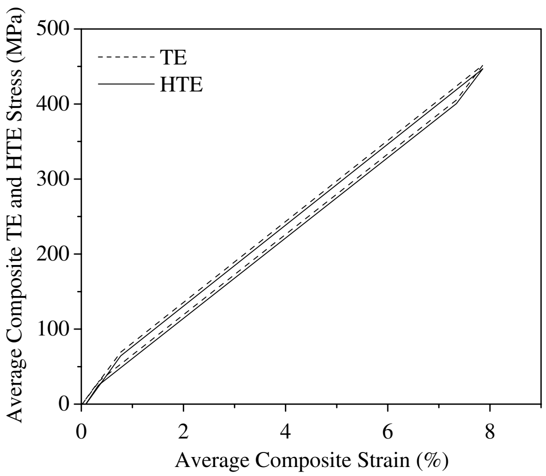

Fig. 8. SMA-composite cell behavior under thermo and hygro-thermo-elastic strain field for $\vartheta^{\Omega} = 0.10$, $\beta_{\mathrm{m}}^{M} = 0.38$, $\chi^{M} = 0.001$ and $\chi^{M} = 0.005$.

The procedure formulated for the composite cell is used to characterize the SMA composite lamina. The inelastic strain tensors ($\overline{P}_{\mathrm{p}}$, $\bar{\lambda}_{\mathrm{t}}$ and $\overline{B}_{\mathrm{m}}$) are used to obtain the phase transformation constant ($\overline{P}$), the hygro-thermo-elastic constant ($\overline{\Lambda}$) and thermo-elastic constant ($\bar{\lambda}$) for the composite lamina, which are given by Eqs. (51a), (61a) and (66a), respectively. A unidirectional lamina under plane stress condition is assumed to investigate the stress–strain behavior and associated hysteresis due to the phase transformation strain in the SMA wire, thermal strain in the SMA wire and matrix, and hygric strain in the matrix. The coupled elastic strain in the composite lamina is computed by using Eq. (71) and the reduced stiffness ($\overline{Q}_{11}^{*}$) for unidirectional loading is computed using Eq. (74). The average composite stresses are evaluated from the constitutive law given in Eq. (73). Analytical results for the on-axis reduced stiffness of SMA composite lamina for different SMA volume fractions are presented in Table 9. The thermo-elastic and hygro-thermo-elastic transformation stresses of the composite lamina for austenite to martensite phase change are given in Table 10 and for martensite to austenite phase change are presented in Table 11. In order to asses the efficiency of the present analytical procedure, computations have been carried out for low ($\vartheta^{\Omega} = 0.05$) and high ($\vartheta^{\Omega} = 0.10$) SMA volume fractions with low and high moisture strains, and the results are shown in Figs. 9 and 10. In Fig. 10, the TE and HTE SMA composite response is represented for temperature of 60 °C and high hygric strain of 0.01, respectively. The results clearly prove the critical importance of introducing moisture for predicting the SMA composite stresses for practical applications. Further, in order to achieve tailored phase transformation

Table 9
Overall on-axis reduced stiffness [MPa] of SMA composite lamina for different SMA wire volume fractions

ϑ^{Ω}	0.01	0.036	0.05	0.10
$\overline{Q}_{11}^{*}$	4115.52	5845.85	6777.57	10105.10

Table 10
Average transformation stresses [MPa] of the SMA composite lamina under unidirectional loading: thermo-elastic and hygro-thermo-elastic strain field

Present approach austenite to martensite transformation		ϑ^{Ω}	0.01	0.036	0.05	0.10
Thermo-elastic stress	$\bar{\sigma}_{\mathrm{s}}^{\mathrm{AS}}$		31.75	45.10	52.28	77.95
Hygro-thermo-elastic stress	$\bar{\sigma}_{\mathrm{s}}^{\mathrm{AS}}$	$\chi^{\mathrm{M}} = 0.005$	23.10	39.32	47.44	75.03
		$\chi^{\mathrm{M}} = 0.01$	14.45	33.54	42.60	72.11
Thermo-elastic stress	$\bar{\sigma}_{\mathrm{f}}^{\mathrm{AS}}$		272.74	276.83	279.03	286.88
Hygro-thermo-elastic stress	$\bar{\sigma}_{\mathrm{f}}^{\mathrm{AS}}$	$\chi^{\mathrm{M}} = 0.005$	264.09	271.05	274.18	283.96
		$\chi^{\mathrm{M}} = 0.01$	255.45	265.27	269.34	281.04

Table 11
Average transformation stresses [MPa] of the SMA composite lamina under unidirectional unloading: thermo-elastic and hygro-thermo-elastic strain field

Present approach martensite to austenite transformation		ϑ^{Ω}	0.01	0.036	0.05	0.10
Thermo-elastic stress	$\bar{\sigma}_{\mathrm{s}}^{\mathrm{SA}}$		251.46	246.60	243.98	234.62
Hygro-thermo-elastic stress	$\bar{\sigma}_{\mathrm{s}}^{\mathrm{SA}}$	$\chi^{\mathrm{M}} = 0.005$	242.81	240.82	239.14	231.70
		$\chi^{\mathrm{M}} = 0.01$	234.61	235.04	234.29	228.78
Thermo-elastic stress	$\bar{\sigma}_{\mathrm{f}}^{\mathrm{SA}}$		13.64	19.35	22.46	33.49
Hygro-thermo-elastic stress	$\bar{\sigma}_{\mathrm{f}}^{\mathrm{SA}}$	$\chi^{\mathrm{M}} = 0.005$	4.99	13.60	17.62	30.57
		$\chi^{\mathrm{M}} = 0.01$	−3.45	7.82	12.78	27.65

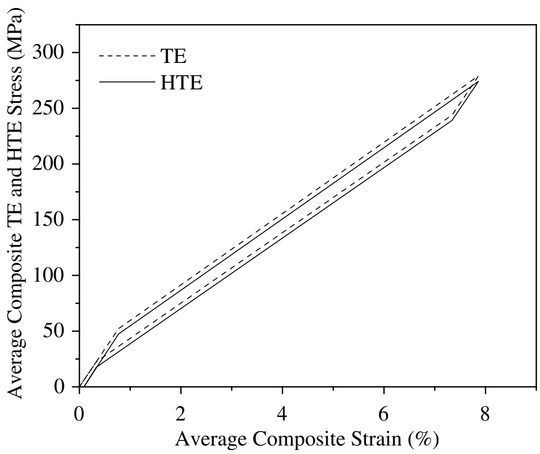

Fig. 9. SMA-composite lamina behavior under thermo and hygro-thermo-elastic strain field for $\vartheta^{\Omega} = 0.05$, $T = 60$ °C, $\beta_{\mathrm{m}}^{M} = 0.38$, $\chi^{M} = 0.001$ and $\chi^{M} = 0.005$.

characteristics, SMA volume fraction must be selected optimally along with the placement of SMA wires in the matrix. Genetic algorithm can be used for this study.

Finally, the characterization of macro-mechanical behaviors of SMA composite needs to be done for different fibre orientations (0–90°) in order to design SMA based smart structures. The variation in the elastic properties with respect to SMA wire orientations is represented in Figs. 11 and 12 for composite cell using Eq. (83). A close observation of Fig. 11 highlights the effect of high Young's modulus and shear modulus

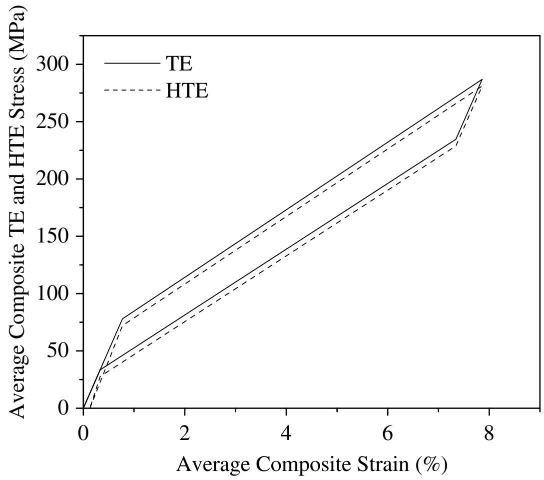

Fig. 10. SMA-composite lamina behavior under thermo and hygro-thermo-elastic strain field for $\vartheta^{\Omega} = 0.10$, $T = 60\,°C$ and $\chi^{M} = 0.01$.

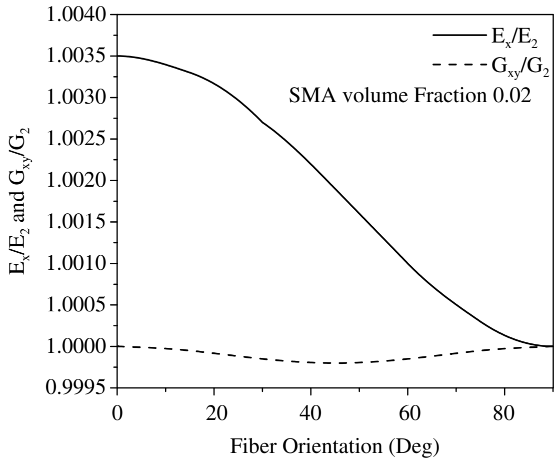

Fig. 11. Variation in Young's modulus and shear modulus for SMA composite cell.

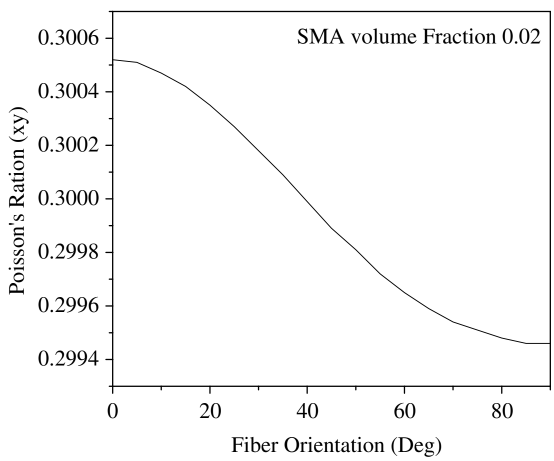

Fig. 12. Variation in Poisson's ratio for SMA composite cell.

of the SMA wire on the off-axis elastic properties of the SMA composite. Similarly, from Fig. 12, it can be observed that the high magnitude of Poison's ratio for the on-axis state (0°) does not reduced significantly for the off-axis (90°) direction of the composite. The on-axis thermal ($\bar{\lambda}$) and moisture ($\overline{\Lambda}$) coefficients ($\beta_{\mathrm{m}}^{M} = 0.38$) are evaluated using the material properties given in Table 2 and the analytical results are listed in Table 12 for different SMA volume fractions. These properties are important because they are computed in terms of material properties as well as the fibre (SMA wire) geometry. The off-axis thermo-elastic coefficients of the lamina are also presented in Fig. 13 for different SMA wire orientations. Similarly, the off-axis elastic and shear modulus for unidirectional SMA lamina can be computed using Eqs. (83) and (84). These characteristics can be used to generate phase transformation behaviors as function of fibre aspect ratio (Eshelby tensor) for different SMA wire orientations to design the SMA composite structures with high stiffness and large strains.

7. Conclusions and applications

The behavior of SMA composite is evaluated under hygrothermal environment using equivalent inclusion method. A one-dimensional SMA wire model is employed to simulate the pseudoelastic behavior in a homogeneous matrix media. The one-dimensional SMA constitutive model, which is redefined in terms of eigen strains (phase transformation strains and thermal strains) in the wire is able to simulate partial and complete phase transformations of SMA wire and SMA composite. It is observed that the SMA volume fraction and hygro-thermo-elastic strain fields influence directly the area of hysterisis loop, transformation stresses and as well as the induced micro-mechanical stresses. The presence of hygrothermal strain significantly changes the stress required for initiating and completing the phase transformation cycle and this will affect the damping capacity of SMA composites. The effect of moisture is more predominant for lower working temperatures (below 60 °C) for which proper matrix properties along with coefficient of moisture expansion must be selected. On-axis and off-axis properties are also obtained for SMA composite to show the high stiffness behavior, which in the present procedure depend not only on material properties but also depend on the fibre (SMA wire) geometry via the Eshelby tensor. The analytical results have been successfully validated with

Table 12
Thermal ($\bar{\alpha}$)[/°C] and Moisture coefficients ($\bar{\beta}$) of SMA lamina

ϑ^{Ω}	0.036	0.05		0.036	0.05		0.036	0.05
$\overline{P}_{11}$	0.43	0.52	$\bar{\alpha}_{11}$	4.09e−005	3.63e−005	$\bar{\beta}_{11}$	0.22	0.18
$\overline{P}_{22}$	0.05	0.07	$\bar{\alpha}_{22}$	5.95e−005	5.78e−005	$\bar{\beta}_{22}$	0.35	0.34
$\overline{P}_{66}$	0.11	0.15	$\bar{\alpha}_{66}$	5.64e−005	5.38e−005	$\bar{\beta}_{66}$	0.32	0.31

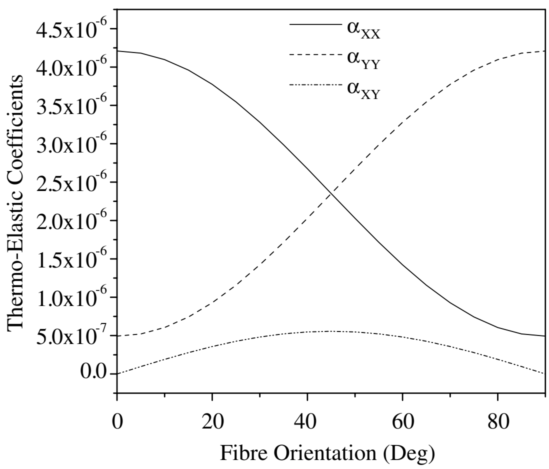

Fig. 13. Off-axis thermo-elastic coefficients for SMA composite lamina for ϑ^{Ω} of 0.036.

results published for the thermo-elastic case in order to prove the ability and efficacy of the modifications and derivations proposed in the procedure. The procedure in this paper can be adopted for;

- Implementation in numerical methods to carry out SMA composite structural analysis.
- To design SMA based smart laminated composites structures for structural applications that include SMA wires periodically embedded in various orientations with different matrix materials.
- To build SMA composites with large diameter SMA/ferromagnetic fibres.
- In the design and experimental validation of the SMA lamina/laminate smart structures under conditions subjected to different (hygro-thermo-elastic) strain fields.
- Different matrix materials (glass epoxy, graphite epoxy) can be used in the analytical procedure.
- For high strain applications, thermo-plastic matrix can also be used as an effective matrix in the SMA composite.

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